Chapter (6) Logarithmic and Exponential functions

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1. It is given that $log_4 x = p$. Giving your answer in its simplest form, find, in terms of *p*,

   a. $log_4(16x) = lg_4 16 + lg_4 x$

   $= lg_4 4^2 + p$ [2]

   $= 2 + p$

   b. $log_4(\frac{x^7}{256}) = lg_4 x^7 - lg_4 256$

   $= 7p - lg_4 4^4$ [2]

   $= 7p - 4$

Using your answers to **parts (i)** and **(ii)**,

   c. solve $log_4(16x) - log_4(\frac{x^7}{256}) = 5$, giving your answer correct to 2 decimal places.

   $2 + p - (7p - 4) = 5$ [3]

   $2 + p - 7p + 4 = 5$

   $-6p + 6 = 5$

   $-6p = -1$

   $p = \frac{1}{6} = 0.17$

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2. The function p is defined by $p(x) = 3e^{x} + 2$ for all real x.
   a. State the range of p.

$y > 2$

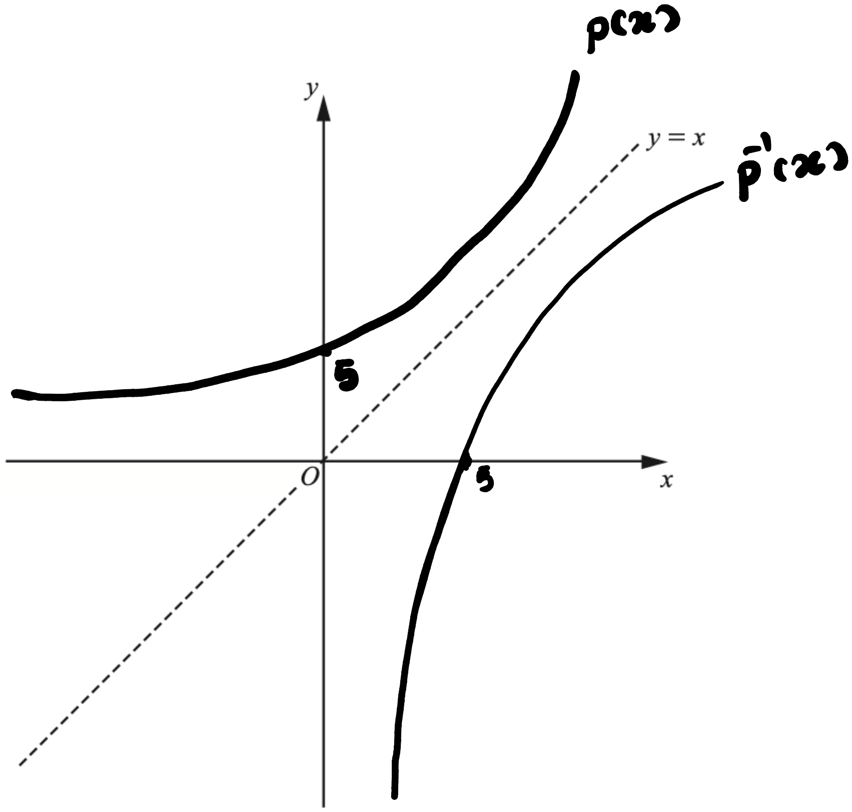
[1]

   b. On the axes below, sketch and label the graphs of $y = p(x)$ and $y = p^{-1}(x)$. State the coordinates of any points of intersection with the coordinate axes.

$y = 3e^{x} + 2$

$x = 0,\ y = 3 + 2$
$= 5$

[3]



   c. Hence explain why the equation $p^{-1}(x) = p(x)$ has no solutions.

because there is no intersection point.

[1]

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3. (a) Solve $log_3 x + log_9 x = 12$.

[3]

$$lg_3 x + lg_{3^2} x = 12$$

$$lg_3 x + \frac{1}{2} lg_3 x = 12$$

$$\frac{3}{2} lg_3 x = 12$$

$$lg_3 x = \overset{4}{\cancel{12}} \times \frac{2}{\cancel{3}}$$

$$lg_3 x = 8$$

$$x = 3^8 = 6561$$

(b) Solve $log_4(3y^2 - 10) = 2log_4(y-1) + \frac{1}{2}$.

[5]

$$lg_4(3y^2-10) - lg_4(y-1)^2 = \frac{1}{2}$$

$$lg_4 \frac{(3y^2-10)}{(y-1)^2} = \frac{1}{2}$$

$$\frac{3y^2-10}{(y-1)^2} = 2$$

$$3y^2 - 10 = 2(y^2 - 2y + 1)$$

$$3y^2 - 10 = 2y^2 - 4y + 2$$

$$y^2 + 4y - 12 = 0$$

$+6$, -2

$$(y+6)(y-2) = 0$$

$$y = -6 \text{ or } y = 2$$

(reject)

4. It is given that $f(x) = 5e^{x} - 1$ for $x \in \mathbb{R}$

a. Write down the range of f.

$y > -1$

[1]

b. Find f^{-1} and state its domain.

[3]

$y = 5e^{x} - 1$

$x = 5e^{y} - 1$

$x + 1 = 5e^{y}$

$\frac{x+1}{5} = e^{y}$

$\ln \frac{x+1}{5} = y$

$f^{-1}(x) = \ln \frac{x+1}{5}, \ x > -1$

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5. $f(x) = e^{3x}$ for $x \in \mathbb{R}$

$g(x) = 2x^{2} + 1$ for $x \geq 0$

a. Write down the range of g.

$g(0) = 1$

$y \geq 1$

[1]

b. Show that $f^{-1}g(\sqrt{62}) = \ln 5$.

[3]

$g(\sqrt{62}) = f(\ln 5)$

$f(\ln 5) = e^{3\ln 5} = 125$

$g(\sqrt{62}) = 2 \times 62 + 1$

$= 124 + 1 = 125$

$\therefore g(\sqrt{62}) = f(\ln 5)$

$f^{-1}g(\sqrt{62}) = \ln 5$

(shown)

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6. Solve $lg(x^2 - 3) = 0$.

[2]

$x^2 - 3 = 10^0$

$x^2 - 3 = 1$

$x^2 = 4$

$x = 2$ or -2

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7. $f(x) = 3e^{2x} + 1$ for $x \in \mathbb{R}$

$g(x) = x + 1$ for $x \in \mathbb{R}$

(a) Write down the range of f and of g.

[2]

$f(x) > 1$

$g(x) \in \mathbb{R}$

(b) Evaluate $fg^2(0)$.

[2]

$fgg(0) = fg(1)$

$= f(2)$

$= 3e^4 + 1$

$= 165$ (3 s.f)

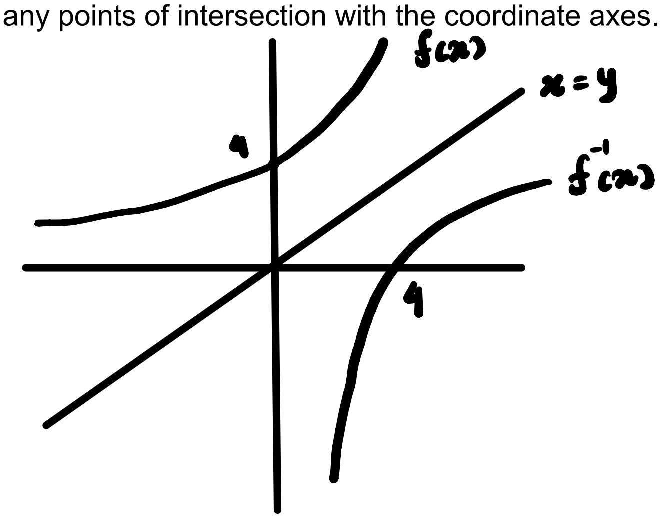
(c) On the axes below, sketch and label the graphs of $y = f(x)$ and $y = f^{-1}(x)$. State the coordinates of any points of intersection with the coordinate axes.

[3]

$y = 3e^{2x} + 1$

$x = 0, y = 3 + 1$

$= 4$



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8. Solve $log_7 x + 2log_x 7 = 3$. [4]

$\lg_7 x + 2\dfrac{\lg_7 7}{\lg_7 x} = 3$

Let $y = \lg_7 x$

$y + \dfrac{2}{y} = 3$

$y^2 - 3y + 2 = 0$

$(y-2)(y-1) = 0$

$y = 2$ or $y = 1$

$\lg_7 x = 2$

$x = 49$

$\lg_7 x = 1$

$x = 7$

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9. (a) Given that $log_a x = p$ and $log_a y = q$, find in terms of p and q.

(i) $log_a axy^2$ [2]

$\lg_a a + \lg_a x + \lg_a y^2$

$= 1 + p + 2q$

(ii) $log_a(\frac{x^3}{ay})$ [2]

$\lg_a x^3 - (\lg_a a + \lg_a y)$

$3p - 1 - q$

(iii) $log_a x + log_y a$. [1]

$p + \dfrac{1}{\lg_a y} = p + \dfrac{1}{q}$

(b) Using the substitution $m = 3^{x}$, or otherwise, solve $3^{x} - 3^{1+2x} + 4 = 0$

[3]

$$3^{x} - 3 \times 3^{2x} + 4 = 0$$

$$m - 3m^{2} + 4 = 0$$

$(\times -1)$

$$3m^{2} - m - 4 = 0$$

$3 \quad -4 \quad 4$

$1 \quad + \quad 1 \quad 3$

$$(3m-4)(m+1) = 0$$

$m = \frac{4}{3}$ or $m = -1$

$3^{x} = -1$ (reject)

$$3^{x} = \frac{4}{3}$$

$$\lg 3^{x} = \lg \frac{4}{3}$$

$$x = 0.262$$